Grab a computer (or take out your tablet/computer)

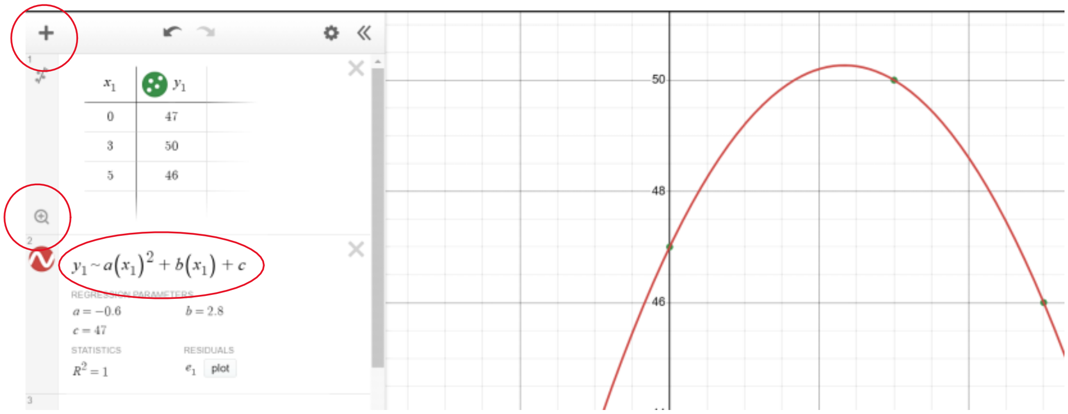
Open Desmos

Open Day 7 Homework (called "FRQ 2 Practice")

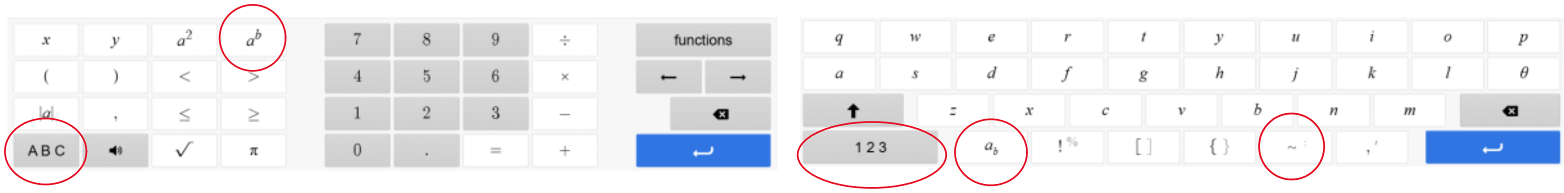
Fun fact: Today is Day 7 and we have a test on Day 9

Part A

- (i) will always ask you to write a system of equations
- (ii) will always ask you to solve the system of equations
- No work needs to be shown
- Could be 2 to 4 parameters
- Round to three decimal places. At least.
- If the units are "thousands" or "millions" or anything like that, you can use 3.4 thousand or use 3400. Either is fine, but be consistent. Treat the number the same throughout the entire problem.



Exponents: Shift + 6
Subscript: Shift + [Key right of 0]
Tilde: Shift + [Key above tab]



Part B: All about average rate of change

- Part (i): Find/calculate AROC.
- Part (ii): Apply/interpret AROC
- Part (iii): Reason using AROC
- The exact type of question can vary, but it'll still fit this format
- Units are not required unless specifically asked for.

Part B(i) - Find Average Rate of Change

Average Rate of Change = Slope

You must show work. The work should look like division of two subtraction problems.

You must write your answer as a decimal. You need at least three numbers after the decimal point unless those numbers are zero (i.e. you can write 4.5 instead of 4.500)

If the units are "thousands" or "millions" or anything like that, you should treat the units the same way you did in part a.

Part B(ii) - Estimating Based on Average Rate of Change

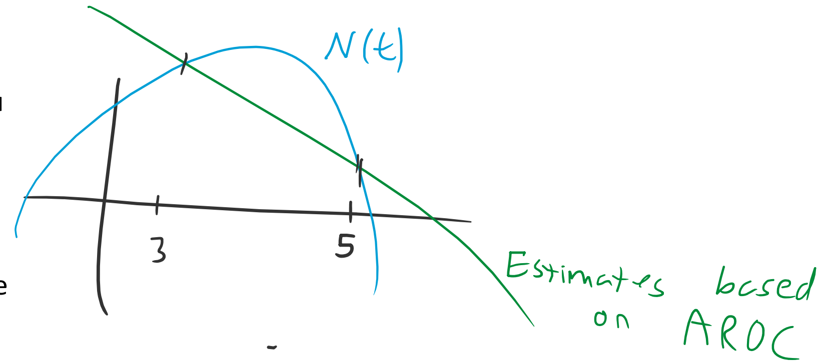
Three Popular Ways to Structure Your Work:

1) Slope Formula = Answer from part B(i)
2) $y - y_1 = m(x - x_1)$ *
3) Arithmetic Sequence

Part B(ii) - Interpret in Context

For "interpret" questions: Use the word "increase" or the word "decrease" to show you understand what a positive or negative rate of change implies. Pair this word with a positive rate of change. Your interpretation should sound like you're describing what d does in an arithmetic sequence. E.g. "From 2002 to 2010, each increase in year corresponded with an increase of $\$20$ in revenue on average."

Part B(iii)



If B(iii) asks you to compare predictions from the model to estimates based on average rate of change, it is asking you to compare a line to the model. Be sure to:

- Draw a picture. Label t-values. Label the line and the curve.
- Mention concavity of the original model
- Say "secant line" to refer to estimates based on average rate of change
- Mention the t-values of the secant line
- Mention the t-values the question is about
- Mention which is on top for the relevant t-values.

We are comparing the concave down function $N(t)$
to outputs from the secant line that passes through N at $t = 3$ and $t = 5$

Since N is concave down, the graph of N is above the graph of the secant line between $t = 3$ and $t = 5$.

So the estimate based on AROC at $t = 4$ is less than the actual value of $N(4)$

Part C: Limitations/Assumptions of the Model

- When is the model appropriate?
- When does the model break down?
- Appropriate domain/range?
- Interpolation is more reliable than extrapolation

You never need to bring in outside knowledge unless it is very obvious.

- Logic you don't need to use unless they explicitly tell you the info:
  - "Video games get less popular because people get tired of playing the same game and want a new one"
  - "4 tons is heavier than a typical car"
  - "People are recycling more, so there should be less trash produced"
- Logic you might need:
  - "The weight of an elephant cannot be negative"
  - "The total number of bikes sold can only increase, never decrease"

Part C: Limitations/Assumptions of the Model

For a question about domain/range restrictions, answer in the form of:
"For [describe t-values], the model __________, but in reality ___________, so we must restrict the [domain/range] to _______."

Examples:

- For $t > 10$, the model has negative output values. In reality, the lowest the weight of the trash pile can be is 0. So, we must restrict the domain to $t \leq 10$.
- For $t > 11$, the model increases. But the question states that in reality sales of lemonade decreased after 2011 ($t = 11$). So, we must restrict our domain to $t \leq 11$.
- For $t < 0$, the model outputs a positive number. But these t-values correspond with a time before cane toads were introduced to the island so in reality the cane toad population was zero at that time. So, we restrict our domain to $t \geq 0$.

$(0, 47)$ $(3, 50)$ $(5, 46)$

FRQ #2 Problem A

t	0	3	5
Number of Ashleys (in thousands)	47	50	46

$t = 1, 2, 4$

$t = 6$

$t = 60$

The table above shows the popularity of the name Ashley among newborns in the United States at different times. In the year 1985 ($t = 0$), 47 thousand American newborns were named Ashley. Three years later ($t = 3$), 50 thousand American newborns were named Ashley. Two years after that ($t = 5$), 46 thousand American newborns were given the name Ashley.

The number of American newborns named Ashley can be modelled by the quadratic function N given by $N(t) = at^2 + bt + c$, where $N(t)$ is the number of newborns named Ashley in thousands t years after 1985.

(A) (i) Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $N(t)$.

(ii) Find the values for a, b, and c as decimal approximations.

A(i)

$47 = a(0)^2 + b(0) + c$

$50 = a(3)^2 + b(3) + c$

$46 = a(5)^2 + b(5) + c$

A(ii)

$a = -0.6$ $\quad$ $b = 2.8$

$c = 47$

(ii) Find the values for a, b, and c as decimal approximations.

(B) (i) Use the given data to find the average rate of change of the number of American newborns named Ashley, in thousands per year, from $t = 3$ to $t = 5$ years. Express your answer as a decimal approximation. Show the computations that lead to your answer.

$(3, 50)$ *

$(5, 46)$

$$\frac{N(5) - N(3)}{5 - 3} = \frac{46 - 50}{5 - 3} = \frac{-4}{2} = -2$$

(ii) Use the average rate of change found in (i) to estimate the number of American newborns named Ashley in 1989 ($t = 4$). Show the work that leads to your answer.

$(4, N(4))$ *

$$\frac{N(4) - 50}{4 - 3} = -2$$

$$\frac{N(4) - 50}{1} = -2$$

$$N(4) - 50 = -2$$

$$\boxed{N(4) = 48}$$

Slope: -2

Point: $(3, 50)$

$y - y_1 = m(x - x_1)$

$y - 50 = -2(x - 3)$

$y = 50 - 2(x - 3)$

$50 - 2(4 - 3)$

$50 - 2(1)$

48

$a_3 = 50$

$a_5 = 46$

$a_4 = ?$

(iii) Compare the estimate found in (ii) to the value given by the model, $N(4)$. Using characteristics of the average rate of change and characteristics of the quadratic model, explain why the two estimates differ.

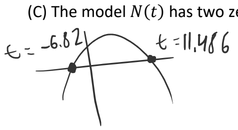
Estimate from AROC: 48

$N(4) = 48.6$

(C) For which t-value, $t = 1$ or $t = 11$, should one have more confidence when using the model N? Give a reason for your answer in the context of the problem.

One should have more confidence using the model to make predictions from $t = 1$ than for $t = 11$ because $t = 1$ is between $t = 0$ and t=5 (interpolation). Whereas, $t = 11$ is far beyond the greatest t-value from our data set (extrapolation).

(C) The model $N(t)$ has two zeros. Explain how these zeros can be used to restrict the domain.



The two zeros suggest we should restrict the domain the t-values between the zeros. Outside of this interval, values of the model are negative which is unrealistic in the context of the problem (the number of newborns named Ashley cannot be negative).

We should restrict the domain to $-6.82 < t < 11.486$ since there cannot be a negative number of newborns named Ashley.

We should restrict the domain to $-6.82 < t < 11.486$ since outside of this interval, $N(t)$is negative.